Quiz 4, Calculus III

Fall 2012

7:24
7:29
8 -2
⇒ 6 min

Name: Key

1. (5 points) Using any method from sections 13.9 or 13.10, find three positive numbers x, y and z such that their sum is 4 and the expression $x + y^2 + z^3$ is a minimum.

$x+y+z=4$ and $x+y^2+z^3 = f(x,y,z)$ is a minimum ✓

$x = 4-y-z \longrightarrow f(y,z) = 4-y-z+y^2+z^3$ ✓

$f_y = -1+2y \qquad f_z = -1+3z^2$

$f_y = 0 \Rightarrow \boxed{y = \frac{1}{2}}$ $\qquad f_z = 0 \Rightarrow \boxed{z = \sqrt{\frac{1}{3}}} \approx 0.6$ ✓✓

$\Rightarrow \boxed{x = 4 - \frac{1}{2} - \frac{1}{\sqrt{3}}}$

Is it a min? $f_{yy} = 2$, $f_{zz} = 6z$, $f_{yz} = 0$

$\Rightarrow d > 0$, $f_{yy} > 0 \Rightarrow$ min.

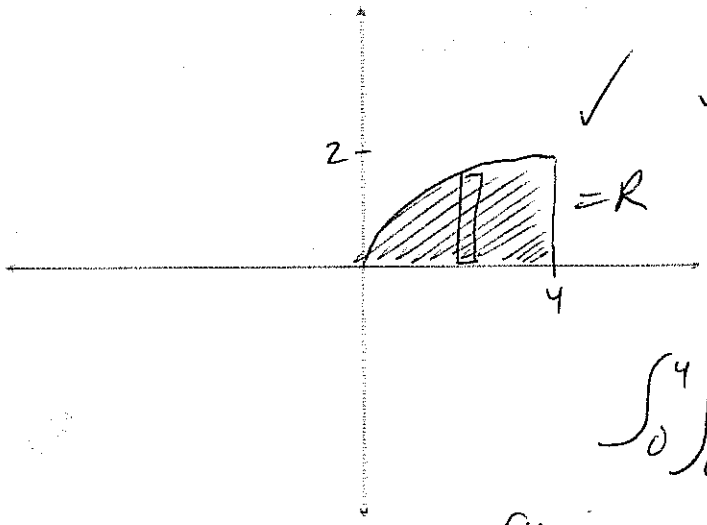
+0.5 extra credit if they check.

2. (5 points) Sketch the region of integration. Then evaluate the iterated integral, switching the order of integration if necessary.

$$\int_0^2 \int_{y^2}^4 \sqrt{x} \sin x \, dx \, dy$$

$y^2 \le x \le 4$

$0 \le y \le 2$



✓ ✓

switch to

$0 \le x \le 4$ ✓

$0 \le y \le \sqrt{x}$

1/2

$$\int_0^4 \int_0^{\sqrt{x}} \sqrt{x} \sin x \, dy \, dx$$

$$= \int_0^4 y(\sqrt{x}\sin x)\Big|_0^{\sqrt{x}} dx$$

1/2

$\int u\,dv = uv - \int v\,du$

$$= \int_0^4 x \sin x \, dx$$

$x = u$ $\quad \sin x \, dx = dv$

$dx = du$ $\quad v = -\cos x$

$$= -x\cos x\Big|_0^4 + \int_0^4 \cos x \, dx$$

$$= -4\cos 4 - 0 + \sin x\Big|_0^4$$

$$= \boxed{-4\cos 4 + \sin 4}$$

$$\approx \boxed{1.86}$$

"Need to know how to integrate by parts (or guess well)
Could be on the Calculator part.